Provincial Report

Grade 9 Mathematics

Achievement Test

October 1984

Student Evaluation

Alberta
EDUCATION

DISTRIBUTION:

Superintendents of Schools
School Principals and Teachers
The Alberta Teachers' Association
Alberta School Trustees' Association
Alberta Education
General Public Upon Request

EXECUTIVE SUMMARY

Findings

The provincial average for the total test was 66.3%.

Student performance on Number Systems was considerably higher than the test average, whereas the performance on Geometry and Algebra was lower than the test average.

Students found it much more difficult to interpret a line graph that has two intersecting lines than to interpret a line graph that has one line.

Students had difficulty with questions involving the total surface area of prisms.

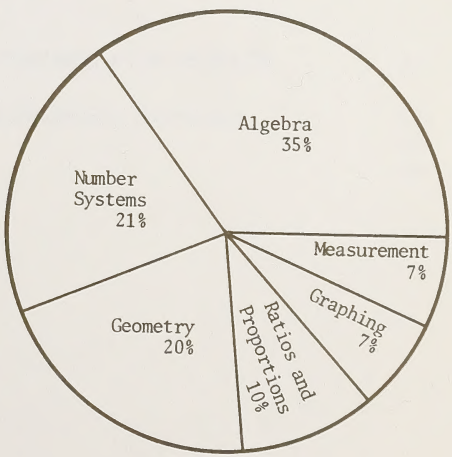
Organization of the Test

The test contains 75 multiple-choice questions covering all six subject strands. The number of questions related to each subject strand reflects the proportion of class time spent on that strand. The emphasis given to each strand is presented below.

Number Systems	16 questions
Ratios and Proportions	8 questions
Measurement	5 questions
Geometry	15 questions
Graphing	5 questions
Algebra	26 questions

Question 29 of the Measurement strand and question 43 of the Geometry strand were deleted from the test because the figures cannot be physically constructed from the given measures.

The relative emphasis given to each strand is shown on the pie chart to the right.



Subject Strand Results

The questions on the test are grouped according to the subject strands. The provincial averages in raw scores for each strand and the total test are:

Number Systems	11.7
Ratios and Proportions	5.6
Measurement	2.8
Geometry	8.8
Graphing	3.3
Algebra	16.4
Total Test	48.4

Comparison with the 1978 MACOSA Results

In 1978, the Minister's Advisory Committee on Student Achievement (MACOSA) administered a similar achievement test to Grade 9 Mathematics students. Nine questions from the 1978 test are on the 1984 test. The averages for those nine questions are as follows:

1978	1984
62.0%	69.1%

Comparison with the 1983 Grade 6 Mathematics Results

Seven questions from the 1983 Grade 6 Mathematics Achievement Test are on the 1984 Grade 9 test. The averages for these questions are as follows:

Grade 6	Grade 9
57.8%	77.2%

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ACKNOWLEDGMENTS

The successful administration of the Grade 9 Mathematics Achievement Test was due to the concerted effort of all involved. Success would have been impossible without substantial contributions from many people, particularly the administrators, teachers, and students, who extended their full co-operation.

The advice received from the Test Review Committee regarding design, development, and reporting has been particularly valuable in the implementation of the Achievement Testing Program. This Committee has representation from:

The Alberta Teachers' Association
The Conference of Alberta School Superintendents
The Universities
Alberta Education

The contribution made by this group is gratefully acknowledged.

The technical expertise provided by Dr. T. O. Maguire, Professor, Division of Educational Research Services, University of Alberta, has also contributed greatly to the advancement of the Achievement Testing Program, and his work in this area is acknowledged and appreciated.

Lloyd E. Symyrozum
Director
Student Evaluation Branch

Chapter 1

THE ACHIEVEMENT TESTING PROGRAM

The purpose of the Achievement Testing Program is to provide educators, trustees, and others with information, significant at the provincial and local levels, about student knowledge, understanding, and skills in relation to program objectives.

The achievement tests are specific to the program of studies prescribed by the Minister of Education. Curriculum specifications for each subject area, provided by the Curriculum Branch and the Language Services Branch of Alberta Education, identify the major content areas, the specific learning objectives within each area, and the emphasis that each objective is to receive. The test questions reflect these curriculum specifications.

The achievement tests are administered on a cyclical basis in four subject areas: language arts, social studies, mathematics, and science, and at three grade levels: 3, 6, and 9. In 1984, achievement tests were administered in Grade 3 Social Studies, Grade 6 English Language Arts, and Grade 9 Mathematics.

Following the achievement test administration in June of each year, the results are reported to each school jurisdiction. These district profiles include results for each school and each student, but individual statements of results are not issued to students.

This provincial report is designed to assist school jurisdictions in interpreting their achievement test results.

Exemptions from the Achievement Testing Program

Under normal circumstances, the following are exempt from achievement testing:

- Students for whom grants are received from the Special Educational Services Branch
- Students in classes in which the subject being tested has been cycled and taught in an alternate year
- Students in classes in which the subject being tested has been taught in an alternate semester
- Students enrolled in English as a Second Language programs for whom grants are received under Section 54(2) of the *School Grants Regulations*

Chapter 2

DESCRIPTION OF TEST AND SAMPLE

This chapter outlines the procedures that were followed during test development, describes the test, and describes the student sample that was used.

Test Development

There were three stages in the development of the Grade 9 Mathematics Achievement Test: preparation of curriculum specifications, development of questions, and selection of questions for the final version of the test.

1. Curriculum Specifications

   The Curriculum Branch prepared curriculum specifications based on the *Program of Studies for Junior High Schools*. These specifications assigned weightings to each major subject strand and to specific topics outlined in the *Program of Studies*. These weightings indicated the emphasis that each topic was to receive in the program. The curriculum specifications were distributed to all school jurisdictions in the province and minor revisions were made on the basis of the feedback that was received. Topic statements from the curriculum specifications upon which specific questions were based are listed under each subject strand with the results in Chapter 3.

2. Development of Questions

   Committees composed of teachers and Student Evaluation Branch personnel constructed questions to reflect the topic statements listed in the curriculum specifications. Questions were also selected from existing tests and item banks. Another teacher committee examined the questions for content validity. Revisions were made on the basis of teacher recommendations and field-test results.

3. Final Test Form

   A draft of the final test was constructed from those questions that best reflected curricular intent and test design requirements. The Test Review Committee reviewed the draft for content validity, accuracy and technical merit. Final changes were made to the test, taking into consideration the recommendations of this committee.

Test Description

The test contains 75 multiple-choice questions. It covers six subject strands: number systems, ratios and proportions, measurement, geometry, graphing, and algebra. The specific content objectives tested within each strand are identified in Chapter 3.

The questions measure four taxonomic levels: knowledge, comprehension, application, and open search. Knowledge questions test recognition of facts or measure ability to perform routine computations. Comprehension questions measure ability to perform algorithms, to translate and to interpret data, and to measure understanding of principles and concepts. Application questions measure ability to solve typical problems, to formulate the problem symbolically, and to manipulate the symbolic representation according to some learned algorithms. Open search questions measure ability to solve novel problems, to prove new results, and to generate new algorithms.

The taxonomic classification of test questions depends on the manner in which the content has been covered in the classroom. A question that is an application question for one class may be a knowledge question for another class. Teachers should examine this classification to determine whether it is consistent with the way they taught the content. The classification of questions is presented in Table 1.

Table 1

Classification of Test Questions

Strand	Number of Items	Test Emphasis	Test Question Numbers by Taxonomic Level			
			Knowledge	Comprehension	Application	Open Search
Number Systems	16	21%	1,2,3,7, 10,11	5,6,8,9,16	4,12,15	13,14
Ratios and Proportions	8	10%	17,18	23	19,20,21,22, 24	--
Measurement	5	7%	25	26	27,28,29	--
Geometry	15	20%	30,36,37	31,34,35,39	32,38,40,41, 42,43,44	33
Graphing	5	7%	--	45	46,47,49	48
Algebra	26	35%	60,62,63, 65	50,51,55,56,61, 64,66,67,68,69, 70,71,72,74,75	53,54,57,58, 73	52,59
Total	75	100%	16	27	26	6

NOTE: Question 29 of the Measurement strand and question 43 of the Geometry strand were deleted from the test because the figures cannot be physically constructed from the given measures. Consequently, the analysis of test results is based on 73 questions.

Population and Sample

A total of 27 831 students was tested (27 121 from 558 public and separate schools and 710 from 65 private schools). The results for students from private schools are not included in the results presented in this report.

The larger school jurisdictions could choose to test either all Grade 9 Mathematics students or only students in randomly selected schools. School boards wanting to sample student achievement were required to notify the Student Evaluation Branch. Only one jurisdiction opted for sampling.

The standard error in the provincial means due to sampling in the one jurisdiction is negligible.

Chapter 3

RESULTS

The report is based on the results for the 27 121 students from public and separate schools. In computing provincial averages, the results from the one jurisdiction that used sampling were weighted to reflect the proportion of the students who were tested in this district.

Results for the Total Test

Since over 98% of the students completed the test, it was concluded that sufficient time was allowed to write the test. Test statistics are as follows:

Provincial Average	66.3%
Standard Deviation	17.8%
KR-20 Coefficient	0.92

Standard-Setting

While provincial averages are useful for comparing the scores of students in a particular school or jurisdiction with overall levels of achievement, it is not possible to know whether the students in the province did as well as they should. A test score by itself has limited meaning without comparison to a standard. Tests vary in difficulty: a raw score of 25 out of 50 for example, could represent very high achievement on one test, and very low achievement on another.

To establish a standard that allows the assessment of achievement on the test, the Student Evaluation Branch followed certain procedures. Twenty experienced Grade 9 Mathematics teachers examined each question on the test and determined the expected difficulty level of that question for three groups of students: borderline passing students, average students, and borderline honors students. From the individual question difficulty levels, the overall test difficulty levels expected for borderline passing students and borderline honors students were determined. The average of the test difficulty levels established by the teachers provided the standards for minimum performance and honors performance on the test. For the Grade 9 Mathematics Achievement Test the standards established were as follows:

> To meet the minimum performance level, students should achieve a score of 50%.
>
> To meet the minimum honors performance level, students should achieve a score of 86%.

From the data gathered for average students, the expected average performance level was established at 66%, which closely approximates the actual provincial average of 66.3%.

Results for Subject Strands and Taxonomic Levels

The questions have been grouped into subtests according to subject strand and taxonomic level. Provincial averages for these subtests and for the total test were computed and rounded to one decimal. Consequently, the sum of the subtest averages is not exactly the same as the average for the total test. Provincial averages for these subtests and the total test are presented in Table 2.

Table 2

Provincial Averages for Subject Strands, Taxonomic Levels, and Total Test

Subtest	Number of Questions	Average in Raw Score	Standard Deviation In Raw Score
Strands			
Number Systems	16	11.7	3.2
Ratios and Proportions	8	5.6	1.8
Measurement	4	2.8	1.1
Geometry	14	8.8	2.9
Graphing	5	3.3	1.1
Algebra	26	16.4	5.6
Taxonomic Levels			
Knowledge	16	11.5	3.1
Comprehension	27	18.4	5.5
Application	24	15.3	4.6
Open Search	6	3.3	1.1
Total Test	73	48.4	13.1

Cautions

The following cautions should be observed when examining the results for the Grade 9 Mathematics Achievement Test:

1. The findings are limited to those that can be obtained from a pencil-and-paper test. Some skills developed in mathematics cannot be measured by this type of test.

2. The questions on the test cover only a representative sample of the objectives for Grade 9 Mathematics in the *Program of Studies* for *Junior High Schools*.

Total Raw Score Frequency Distribution

Table 3 presents total raw scores as well as the relative frequency and the cumulative frequency for each raw score. The range of scores was from 6 to 73. Any relative frequency or any cumulative frequency smaller than 0.05 was rounded to 0.0. For example, 10 students or 0.04% obtained a score of 13, but the relative frequency for this score is recorded as 0.0.

Table 3

Total Raw Score Frequency Distribution

Total Raw Score	Relative Frequency in %*	Cumulative Frequency in %**	Total Raw Score	Relative Frequency in %*	Cumulative Frequency in %**
6	0.0	0.0	40	2.0	28.6
7	0.0	0.0	41	2.2	30.8
8	0.0	0.0	42	2.3	33.1
9	0.0	0.0	43	2.2	35.3
10	0.0	0.0	44	2.4	37.7
11	0.0	0.0	45	2.5	40.2
12	0.0	0.0	46	2.5	42.7
13	0.0	0.1	47	2.6	45.3
14	0.1	0.2	48	2.4	47.7
15	0.1	0.3	49	2.5	50.2
16	0.1	0.4	50	2.6	52.8
17	0.2	0.6	51	2.6	55.4
18	0.2	0.8	52	2.4	57.8
19	0.4	1.2	53	2.7	60.5
20	0.4	1.6	54	2.7	63.2
21	0.5	2.1	55	2.6	65.8
22	0.5	2.6	56	2.4	68.2
23	0.6	3.2	57	2.7	70.9
24	0.8	4.0	58	2.5	73.4
25	0.9	4.9	59	2.9	76.3
26	1.1	6.0	60	2.4	78.7
27	1.0	7.0	61	2.6	81.3
28	1.1	8.1	62	2.4	83.7
29	1.3	9.4	63	2.2	85.9
30	1.3	10.7	64	2.2	88.1
31	1.5	12.2	65	2.1	90.2
32	1.6	13.8	66	2.1	92.3
33	1.5	15.3	67	1.8	94.1
34	1.7	17.0	68	1.7	95.8
35	1.8	18.8	69	1.5	97.3
36	1.8	20.6	70	1.1	98.4
37	1.9	22.5	71	0.9	99.3
38	2.2	24.7	72	0.5	99.8
39	1.9	26.6	73	0.2	100.0

*Relative frequency: the percentage of students who obtained each score.

**Cumulative frequency: the percentage of students who scored at or below each score.

Results for Individual Questions

The percentage of students choosing each response for each question is given in Table 4. The correct response for each question is also identified.

Table 4

Results for Individual Questions

Item	Key	Distribution of Responses in %*				Item	Key	Distribution of Responses in %*			
		A	B	C	D			A	B	C	D
1	C	0.2	3.5	**95.8**	0.4	39	D	23.3	5.4	8.6	**62.5**
2	A	**63.7**	27.8	2.0	6.4	40	D	19.3	11.2	9.2	**59.7**
3	D	17.4	19.2	5.8	**57.4**	41	C	6.1	19.2	**64.0**	10.3
4	C	7.0	4.9	**83.8**	4.0	42	A	**40.5**	21.3	24.3	13.3
5	D	5.0	4.5	6.1	**83.8**	43	C		Omitted		
6	A	**71.9**	6.4	9.3	12.2	44	B	6.9	**77.4**	9.0	6.4
7	A	**74.7**	7.1	9.9	8.0	45	A	**87.8**	2.7	3.6	5.8
8	B	10.1	**66.3**	6.8	16.7	46	D	5.6	2.6	8.7	**82.9**
9	C	9.8	14.5	**56.1**	18.9	47	A	**54.7**	14.0	23.9	7.0
10	A	**85.4**	4.1	8.2	2.1	48	D	17.2	13.9	24.0	**44.3**
11	C	8.9	9.3	**78.8**	2.8	49	B	5.2	**53.9**	21.2	19.4
12	B	7.6	**66.5**	22.4	3.3	50	C	6.0	10.6	**69.1**	13.9
13	D	4.0	6.7	7.0	**81.5**	51	A	**64.5**	14.5	10.7	10.0
14	B	4.0	**52.9**	22.8	19.5	52	D	7.1	17.5	20.3	**54.5**
15	B	8.7	**73.5**	9.4	8.0	53	D	7.4	3.9	5.8	**82.5**
16	D	12.0	1.6	8.1	**78.2**	54	A	**61.6**	18.6	15.8	3.6
17	C	7.3	6.1	**78.9**	7.5	55	C	8.2	48.1	**40.5**	2.9
18	D	11.2	22.3	8.5	**57.8**	56	B	27.3	**53.8**	16.8	1.8
19	B	23.9	**62.9**	4.7	8.3	57	A	**30.2**	25.8	38.2	5.5
20	D	1.6	2.9	2.6	**92.5**	58	B	14.7	**55.4**	9.3	20.2
21	A	**60.1**	22.4	7.9	9.4	59	C	10.5	26.6	**53.7**	8.3
22	B	16.7	**58.6**	22.2	2.2	60	A	**41.4**	43.6	7.1	7.5
23	D	2.0	13.5	6.1	**78.1**	61	B	7.5	**77.4**	9.1	5.5
24	C	14.2	6.9	**69.7**	8.8	62	C	10.3	12.4	**69.0**	8.0
25	C	7.2	3.3	**80.6**	8.8	63	D	12.8	14.9	7.3	**64.5**
26	B	15.8	**69.5**	6.7	7.8	64	C	39.6	9.1	**44.5**	6.4
27	C	19.5	7.1	**68.0**	4.9	65	B	12.4	**78.9**	5.0	3.2
28	C	8.0	24.2	56.8	9.9	66	C	4.1	6.9	**77.8**	10.6
29	B		Omitted			67	D	8.1	9.6	18.6	**63.0**
30	A	**81.3**	6.8	2.9	8.7	68	B	7.2	**85.5**	3.9	2.9
31	B	6.1	**86.8**	4.3	2.7	69	A	**81.0**	8.5	6.2	3.8
32	B	10.0	**52.8**	30.6	6.2	70	B	14.2	**49.9**	12.4	23.0
33	D	9.6	18.6	30.2	**41.0**	71	A	**65.6**	10.6	9.5	13.6
34	C	7.7	19.5	**64.0**	8.1	72	C	11.2	19.5	**59.9**	8.8
35	D	16.3	14.1	13.3	**56.1**	73	A	**62.0**	13.1	15.2	8.8
36	D	11.4	2.3	9.3	**76.8**	74	D	8.1	18.7	10.6	**61.7**
37	A	**57.6**	10.1	16.2	15.7	75	A	**83.2**	4.9	6.1	4.6
38	A	**51.6**	34.4	8.8	4.9						

*The sum of the percentages for each question may be less than 100% because the No Response category is not included. This category is less than 2% for all questions.

Discussion of Results for Subject Strands

The results for each subject strand are discussed in detail in the following sections. The topics that were tested within each strand are identified. The easiest and the most difficult questions within each subject strand are noted. Sample questions from the test are provided. For each sample question, the curriculum objective and the taxonomic level are identified. The asterisk (*) indicates the correct response for each question, and the percentage of students who selected each alternative is given.

Number Systems (Questions 1 to 16)

Questions related to this strand measure ability to

- write the values for powers
- understand and use the properties $(a^x)^y = a^{xy}$ and $a^{-x} = 1/a^x$
- solve word problems involving multiplication and subtraction
- express a number as a product of prime factors
- simplify expressions involving the order of arithmetic operations
- write numbers in scientific notation
- add and subtract negative rational numbers
- change positive rationals in the form a/b to decimals
- change rational numbers in decimal form to the form a/b
- solve problems involving positive and/or negative rationals
- estimate products
- estimate square roots of numbers

- The average score for the 16 questions on number systems was 73.2%.

- Question 1, requiring students to write the value for powers, was found to be the easiest (95.8% answered correctly).

- Question 14, requiring students to solve a problem involving addition and subtraction of positive rationals, was found to be the most difficult (52.9% answered correctly).

Sample questions:

Question 8
Curriculum Objective: Add negative rational numbers
Taxonomic Level: Comprehension

8. The value of $-\frac{4}{5} + \left(-\frac{3}{4}\right)$ is

		Student Responses
A.	$-\frac{7}{9}$	10.1%
*B.	$-1\frac{11}{20}$	66.3%
C.	$\frac{3}{5}$	6.8%
D.	$1\frac{11}{20}$	16.7%

Alternative A, arrived at by adding the numerators and the denominators of the two fractions, attracted 10% of the students. Alternative C, arrived at by approaching the question as if it were a multiplication problem and cancelling the 4's, attracted 7% of the students. Alternative D attracted 17% of the students. These students may have disregarded the negative signs or they may have thought that the sum of two negatives equals a positive just as the multiplication or division of two negatives equals a positive.

Question 12
Curriculum Objective: Solve problems involving negative rationals
Taxonomic Level: Application

12. The sum of two rational numbers is -4.83. If one of the numbers is -2.83, then the other number is

		Student Responses
A.	-7.66	7.6%
*B.	-2.00	66.5%
C.	2.00	22.4%
D.	7.66	3.3%

Alternatives A and D, arrived at by adding the absolute values of the rational numbers and confusing the final sign, attracted 11% of the students. These students may have neglected the signs in this question or they may have thought that the sum or difference of two negatives equals a positive just as the multiplication or division of two negatives equals a positive.

Ratios and Proportions (Questions 17 to 24)

Questions related to this strand measure ability to

- express a per cent as a fraction
- express a decimal as a per cent
- use ratios to solve problems involving interest, discounts, tax, etc.
- use ratios in scale drawings

- The average score for the eight questions on ratios and proportions was 70.1%.

- Question 20, requiring students to solve a problem involving ratios, was found to be the easiest (92.5% of students answered correctly).

- Question 18, requiring students to express 0.7 as a per cent, was found to be the most difficult (57.8% answered correctly).

Sample questions:

Question 18
Curriculum Objective: Maintain previously developed skills (express a decimal as a per cent)
Taxonomic Level: Knowledge

18. 0.7 expressed as a per cent is

		Student Responses
A.	0.7%	11.2%
B.	7%	22.3%
C.	$\frac{7}{10}\%$	8.5%
*D.	70%	57.8%

This question proved to be more difficult than was expected, since this content objective was first introduced in Grade 6. Alternatives A, B, and C reflect common errors made by students. Field-test studies indicate that students have little difficulty converting two-decimal numerals to per cents (e.g., 0.65 = 65%). Since this question involves a one-digit numeral, students may have failed to understand that the decimal must be moved two places to the right regardless of the number of digits in the numeral.

Question 22
Curriculum Objective: Use ratios to solve problems involving discounts
Taxonomic Level: Application

22. A store offered a 15% discount on a television set. If the TV was regularly priced at $550, the sale price was

		Student Responses
A.	$82.50	16.7%
*B.	$467.50	58.6%
C.	$495.00	22.2%
D.	$632.50	2.2%

This question requires students to perform a two-step calculation. Alternative A, arrived at by only calculating the amount of the discount, attracted 17% of the students. Alternative C, arrived at by using 10% discount rather than 15%, attracted 22% of the students. These students may have been reading carelessly or guessing. Alternative D, arrived at by adding the amount of the discount to the regular price, attracted only 2% of the students. Students realized that the sale price cannot be more than the regular price.

Measurement (Questions 25 to 29)

Questions related to this strand measure ability to

- express equivalent measures within units of length
- find perimeters and areas of polygons

- The average score for the four questions on measurement was 69.1%.
- Question 25, requiring students to express the number of centimetres in one metre, was found to be the easiest (80.6% answered correctly).
- Question 28, requiring students to find the perimeter of a racetrack whose shape consists of two sides of a rectangle and two half-circles, was found to be the most difficult (56.8% answered correctly).

Sample questions:

Question 26
Curriculum Objective: Maintain previously developed skills (find perimeters of polygons)
Taxonomic Level: Comprehension

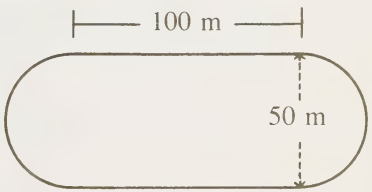
26. The perimeter of a regular hexagon that has a side of 13 m is

		Student Responses
A.	65 m	15.8%
*B.	78 m	69.5%
C.	91 m	6.7%
D.	104 m	7.8%

Alternative A, arrived at by multiplying 13 m by 5 sides (perimeter of a pentagon), attracted 16% of the students. Alternative C, arrived at by multiplying 13 m by 7 sides, attracted 7% of the students. Alternative D, arrived at by multiplying 13 m by 8 sides (perimeter of an octagon), attracted 8% of the students. The students who failed to answer this question correctly likely did not remember the number of sides in a hexagon.

Question 28
Curriculum Objective: Find perimeters of polygons
Taxonomic Level: Application

28. Find the perimeter of the racetrack illustrated at the right. [$\pi = 3.14$]



		Student Responses
A.	3570 m	8.0%
B.	1570 m	24.2%
*C.	357 m	56.8%
D.	157 m	9.9%

The students must realize that the perimeter of the racetrack consists of two sides of a rectangle and two half-circles.

Alternative A, arrived at by adding the digit zero to the correct perimeter, attracted 8% of the students. Alternative B, arrived at by adding the digit zero to the circumference of the circle, attracted 24% of the students. Students choosing alternatives A and B may have been influenced by the zeros in the diagram. Alternative D, arrived at by finding the circumference of the circle, attracted 10% of the students. Since the correct answer is the only reasonable answer, it appears that students who chose alternatives A, B, and D failed to check their answers by estimating a reasonable answer from the diagram.

Geometry (Questions 30 to 44)

Questions related to this strand measure ability to

- define an acute angle
- find the measure of an angle from the vertical angle and the complementary angle
- solve a problem involving the Theorem of Pythagoras
- test congruency using SAS
- construct a regular hexagon
- identify an edge of a prism
- develop formulas to measure volume and surface area of right prisms
- solve problems involving the lateral surface area of a can, the volumes of a can and a right prism, and the total surface areas of a cube and a right prism

- The average score for the 14 questions on geometry was 62.5%.

- Question 31, requiring students to find the measure of an angle from the vertical angle and the complementary angle, was found to be the easiest (86.8% answered correctly).

- Question 42, requiring students to find the total surface area of a cube, was found to be the most difficult (40.5% answered correctly).

Sample questions:

Question 32
Curriculum Objective: Solve a problem involving the Theorem of Pythagoras
Taxonomic Level: Application

32. The length of a rectangle is 12 m and the width is 9 m. The measure of the diagonal is

		Student Responses
A.	3 m	10.0%
*B.	15 m	52.8%
C.	21 m	30.6%
D.	225 m	6.2%

Alternative A, arrived at by subtracting 9 from 12 or by finding the square root of 9, attracted 10% of the students. Alternative C, arrived at by adding 12 and 9, attracted 31% of the students. Alternative D, arrived at by adding 12^2 and 9^2, was chosen by 6% of the students. It would appear that students who chose A or C did not relate the Theorem of Pythagoras to solving the question, or that they may have performed the simplest mathematical computations in an effort to complete the test quickly.

Question 38
Curriculum Objective: Develop a formula to measure the surface area of right prisms
Taxonomic Level: Application

38. The formula for the total surface area of a rectangular right prism, whose dimensions are ℓ, w, h, is

		Student Responses
*A.	$A_s = 2\ell w + 2wh + 2\ell h$	51.6%
B.	$A_s = 2\ell + 2w + 2h$	34.4%
C.	$A_s = \ell w + wh$	8.8%
D.	$A_s = \ell w$	4.9%

Alternative B, arrived at by doubling each of the dimensions given in the stem and adding them together, attracted 34% of the students. Alternative C, arrived at by finding the areas of two faces, attracted 9% of the students. Alternative D, arrived at by finding the area of one face, attracted 5% of the students. Students most likely would have already seen or used the formula they were required to develop. It appears that they had difficulty visualizing or drawing a three-dimensional object and relating the symbols to a real rectangular right prism.

Question 41
Curriculum Objective: Solve problems involving the volume of a can
Taxonomic Level: Application

41. Given the formula $V = \pi r^2 h$, the volume of a can that has an inside diameter of 6 cm and a height of 7 cm is [$\pi = 3.14$]

		Student Responses
A.	65.94 cm^3	6.1%
B.	131.88 cm^3	19.2%
*C.	197.82 cm^3	64.0%
D.	791.28 cm^3	10.3%

This question requires students to substitute numbers into a formula. It was expected that students would find this question easier because the formula is given. Alternative A, arrived at by using πrh, attracted 6% of the students. Alternative B, arrived at by using $2\pi rh$ or πdh, attracted 19% of the students. Alternative D, arrived at by using πd^2h, attracted 10% of the students. It would appear that the students who chose alternatives A, B, and D did not understand what the symbols in the formula represent.

Question 42
Curriculum Objective: Solve problems involving the total surface area of a cube
Taxonomic Level: Application

42. The measure of an edge of a cube is 5 cm. The total surface area of the cube is

		Student Responses
*A.	150 cm^2	40.5%
B.	125 cm^2	21.3%
C.	30 cm^2	24.3%
D.	25 cm^2	13.3%

Alternative B, arrived at by finding the volume of the cube or finding the areas of five faces of the cube, attracted 21% of the students. Students choosing B may have confused the terms "volume" and "surface area," or may have failed to realize that a cube has six faces. Alternative C, arrived at by multiplying the measure of one edge by six faces, attracted 24% of the students. Students who chose C recognized the fact that a cube has six faces, but may have failed to read the question carefully. Alternative D, arrived at by finding the area of one face only, attracted 13% of the students. Students who chose D may have forgotten that the question asks for the total surface area of a cube.

Graphing (Questions 45 to 49)

Questions related to this strand measure ability to

- generate ordered pairs and make graphs from the ordered pairs
- interpret graphs
- read approximate square roots of non-perfect square numbers from the graph

- The average score for the five questions on graphing was 64.9%.

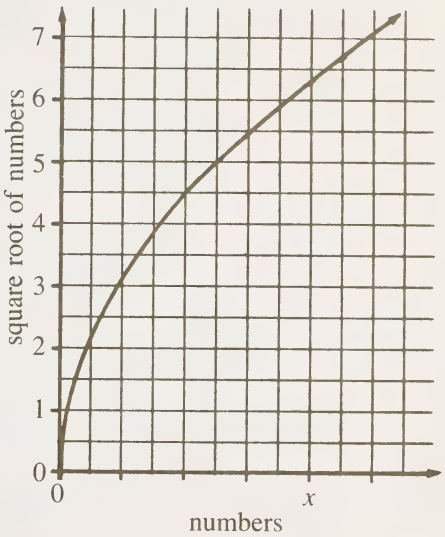
- Question 45, requiring students to interpret a line graph that has one line, was found to be the easiest (87.8% answered correctly).

- Question 48, requiring students to interpret a line graph that has two intersecting lines, was found to be the most difficult (44.3% answered correctly).

Sample question:

Question 49
Curriculum Objective: Read approximate square roots of non-perfect square numbers from the graph
Taxonomic Level: Application

49. From the graph at the right, if x represents a number, then the best estimate of $\sqrt{x}$ is



		Student Responses
A.	6.1	5.2%
*B.	6.3	53.9%
C.	6.5	21.2%
D.	6.7	19.4%

The question was carefully designed so that students with calculators could not use them but had to read the graph to arrive at the answer. The graph was accurately drawn so that alternative B was clearly correct. Students who chose alternatives A, C, or D failed to read the graph carefully or did not interpret the units on the vertical axis correctly.

Algebra (Questions 50 to 75)

Questions related to this strand measure ability to

- solve first degree equations in one variable
- solve a problem by writing an equation in one variable and solving the equation
- interpret mathematical data and express it as a relationship
- predict the effect of altering specific elements of formulas
- solve problems that require the use of formulas
- identify a constant term
- evaluate expressions
- classify and identify a binomial
- write the polynomial in standard form
- translate an English expression into an algebraic expression
- identify the numerical coefficient of a monomial
- combine like terms
- find the sum of polynomials
- find the quotient of monomials
- find the product of a monomial and a polynomial
- factor polynomials by taking out the greatest common factors
- find the products of binomials
- factor trinomials

- The average score for the 26 questions on algebra was 62.9%.

- Question 68, requiring students to find a quotient of monomials, was found to be the easiest (85.5% answered correctly).

- Question 57, requiring students to find the time it takes to travel x kilometres given the speed of the train, was found to be the most difficult (30.2% answered correctly).

Sample questions:

Question 52
Curriculum Objective: Solve a problem by writing an equation in one variable and solving the equation
Taxonomic Level: Open search

52. A 170 cm board is cut into 3 pieces. One piece is 25 cm longer than another piece, and the longest piece is equal to the sum of the lengths of those two pieces. What is the length of the shortest piece?

		Student Responses
A.	85 cm	7.1%
B.	60 cm	17.5%
C.	55 cm	20.3%
*D.	30 cm	54.5%

This question requires students to apply a multi-step problem-solving approach as follows:

Step 1 Use a variable to determine the length of the shortest piece and then express the lengths of the other two pieces in relation to the first piece:

$$x,\ x + 25,\ 2x + 25$$

Step 2 Write the equation. Students must realize that the sum of the three pieces equals the length of the board:

$$x + x + 25 + 2x + 25 = 170$$
$$4x + 50 = 170$$

Step 3 Solve the equation correctly:

$$4x + 50 = 170$$
$$4x = 170 - 50 = 120$$
$$x = 30$$

Students who chose alternative A may have solved for x correctly and then found the length of the longest piece. Students who chose alternative B may have expressed the lengths of three pieces as x, 25, and $x + 25$ and then set up the equation and solved for x:

$$x + 25 + x + 25 = 170$$
$$2x + 50 = 170$$
$$2x = 170 - 50 = 120$$
$$x = 60$$

Students who chose alternative C may have made a mistake while solving Step 3:

$$4x + 50 = 170$$
$$4x = 170 + 50 = 220$$
$$x = 55$$

or they may have solved for x correctly and then added 25 (from the stem) to 30. The result is the length of the middle piece.

This problem is one of the most complex questions on the test. Even though only 55% of students answered this question correctly, the result is quite satisfactory.

Question 55
Curriculum Objective: Predict the effect of altering specific elements of formulas
Taxonomic Level: Comprehension

55. In the formula $d = vt$, if v is tripled and t is doubled, d will be

		Student Responses
A.	three times as large	8.2%
B.	five times as large	48.1%
*C.	six times as large	40.5%
D.	nine times as large	2.9%

Alternative A, arrived at by considering only the tripling of v, attracted 8% of the students. Alternative B, arrived at by adding 3 and 2, attracted 48% of the students. Alternative D, arrived at by squaring 3, attracted 3% of the students. Students appeared to have difficulty applying the operations of tripling and doubling to a relationship that is presented symbolically rather than numerically.

Question 60
Curriculum Objective: Identify a constant term
Taxonomic Level: Knowledge

60. In the expression $2 + x + 3x + x^2$, which term is the constant term?

		Student Responses
*A.	2	41.4%
B.	x	43.6%
C.	$3x$	7.1%
D.	x^2	7.5%

This question measures student knowledge of basic algebraic terminology. However, over half of the students chose incorrect alternatives. It appears that students were unfamiliar with the algebraic terms "constant" and "variable."

Question 70
Curriculum Objective: Factor polynomials by taking out the greatest common factors
Taxonomic Level: Comprehension

70. When $4x^2 - 4x + 4$ is factored, the answer is

		Student Responses
A.	$4(x^2 - x)$	14.2%
*B.	$4(x^2 - x + 1)$	49.9%
C.	$4x^2(-4x + 4)$	12.4%
D.	$(2x - 2)(2x - 2)$	23.0%

It appears that after factoring the polynomial, the students failed to verify their answers.

Comparison with the 1978 MACOSA Results

In 1978, a similar achievement test was administered to Grade 9 Mathematics students in Alberta. Nine questions from the 1978 test are on the 1984 test. The common questions are distributed across three subject strands. The number of questions and the question numbers in each strand, as well as the provincial averages for 1978 and 1984, are presented in Table 5.

Table 5

Provincial Averages for Common Questions: 1978 and 1984

Subject Strand	Number of Questions	Question Numbers	Averages in Per Cent 1978	1984
Number Systems	3	1,4,10	78.4	88.3
Measurement	1	25	72.9	80.6
Algebra	5	54,57,67, 72,74	49.9	55.3
Total	9		62.0	69.1

Comparison with the 1983 Grade 6 Mathematics Results

Seven questions from the 1983 Grade 6 Mathematics Achievement Test are on the Grade 9 test. These questions are distributed across five subject strands. The number of questions and the question numbers in each strand, as well as the provincial averages for Grade 6 and Grade 9, are presented in Table 6.

Table 6

Provincial Averages for Common Questions: Grade 6 and Grade 9

Subject Strand	Number of Questions	Question Numbers	Averages in Per Cent	
			Grade 6	Grade 9
Number Systems	1	13	53.6	81.5
Ratios and Proportions	2	18,20	49.6	75.2
Measurement	2	25,27	61.4	74.3
Geometry	1	44	62.1	77.4
Graphing	1	46	67.1	82.9
Total	7		57.8	77.2

NOTE: Question 25 appeared in both the MACOSA and the Grade 6 tests.

Chapter 4

GUIDE TO THE INTERPRETATION OF JURISDICTION RESULTS

In addition to their use in monitoring student achievement for the province as a whole, the results of the Grade 9 Mathematics Achievement Test are useful in comparing achievement in a particular jurisdiction with provincial results. However, care must be exercised in making these comparisons and in drawing conclusions from the data.

The following jurisdiction and school reports are provided under separate cover for each jurisdiction.

1. The Jurisdiction Summary Report contains jurisdiction results parallel to the provincial results in Table 2 and Table 3.

2. The School Summary Reports contain the school results parallel to the provincial results in Table 2 and Table 3.

3. The Jurisdiction Item Alternative Response Frequency Data contains jurisdiction results parallel to the provincial results in Table 4.

4. The School Item Alternative Response Frequency Data contains school results parallel to the provincial results in Table 4.

5. The Individual Student Subtest Results

These reports are confidential to the jurisdiction.

Differences Between Jurisdiction and Provincial Averages

Jurisdictions are provided with their average scores for each subtest. These scores may be compared to the provincial average for the same subtest. However, the importance of the differences between jurisdiction averages and provincial averages is not always obvious. To aid in the interpretation of differences between the averages, jurisdiction reports indicate when the difference between the jurisdiction average and the provincial average is unlikely to be due to chance variation in the abilities of students. For the purposes of the provincial testing program, the 95% confidence interval is used. That is to say, if the probability is only one in 20 that the difference is due to chance, the jurisdiction average is considered different from the provincial average. Otherwise, it is classified as not different from the provincial average.

A statistical test of significance is employed for each subtest for each jurisdiction. The provincial average for that subtest and the provincial standard deviation for that subtest determine the population average and standard deviation. The standard deviation of a distribution is a measure of the variation of scores. In a normal distribution, there is a fixed and known relationship between the standard deviation and the proportion of individual scores in any part of the distribution. For example, 68% of scores fall within one standard deviation of the mean (average). If a test has a mean of 50 and a standard deviation of 10, 68% of those writing the test scored between 40 and 60.

The amount of chance variation in jurisdiction averages varies with the number of students tested in that jurisdiction. When any random sample is drawn from a population, its average is expected to be the same as the population average. Yet the actual group average may vary because of individual variation in the sample. This variation is known as the error of the mean. The amount of variation in the averages of random samples drawn from the population is related to the standard deviation of the scores in the population. When the population mean and standard deviation are known, as in the case of the achievement tests, it is possible to determine how likely it is that any subgroup, such as a jurisdiction, represents a random sample of the population in achievement. This statistical test, known as a one-sample z-test, is the one that has been applied to jurisdiction scores in each subtest. Thus if a jurisdiction is classified as different from the provincial average, there is less than one chance in 20 that the difference between the average score for the jurisdiction on that subtest and the provincial average would occur in a group of that size selected at random from all students in the province. In other words, the difference is statistically significant at the 0.05 level.

Because these achievement levels are calculated taking jurisdiction size into consideration, two jurisdictions with the same averages but of different sizes may be classified differently. The larger jurisdiction would be more likely to be above or below average, because the amount of chance variation would be less in larger jurisdictions, and the actual difference would represent a larger deviation from the provincial average.

For example, imagine two jurisdictions, A with 25 students writing Test X, and B with 100 students writing Test X. Both jurisdictions have the same average, 54.2. Test X has a provincial average score of 50.0 and a standard deviation of 12.0. The difference between the provincial average and the jurisdiction average is 4.2. A difference this large would be expected 8 times out of 100 for groups of 25 selected at random from the population, and fewer than 3 times out of 1000 for groups of 100. Thus the difference from the provincial average would not be statistically significant for Jurisdiction A, but would be for Jurisdiction B.

When it has been determined that a difference is significant, the direction of the difference is important, particularly for those jurisdictions below the provincial average. These jurisdictions are encouraged to identify the sources of these differences.

Table 7 indicates the percentage of jurisdictions classified as significantly above or below the provincial average for each subtest.

Table 7

Distribution of Jurisdiction Levels of Achievement

Subtest	Distribution of Jurisdictions		
	% Below Provincial Average	% Not Different From Provincial Average	% Above Provincial Average
Total Test	27.5	47.8	24.6
Strands			
Number Systems	22.5	49.3	28.3
Ratios and Proportions	23.2	58.7	18.1
Measurement	28.3	55.1	16.7
Geometry	32.6	45.7	21.7
Graphing	26.1	68.8	5.1
Algebra	27.5	50.0	22.5
Taxonomic Levels			
Knowledge	27.5	52.9	19.6
Comprehension	22.5	48.6	29.0
Application	28.3	52.2	19.6
Open Search	18.1	63.8	18.1

In examining the test results, the reader must keep in mind that a test score does not indicate why a particular performance occurred, but only that it did occur. After studying the results, the identification of reasons for that performance should be undertaken. There are a variety of factors that should be examined:

1. Student motivation. Consideration should be given to the degree to which students were motivated to perform to their levels of ability.

2. Student ability. While the statistical test of significance is designed to take into consideration fluctuations in the average ability levels of students, it is possible that a group of students with a particularly high or low average ability may come through a system. This is much more likely to be a factor in small systems than in large systems.

3. Teaching and curriculum. Consideration should be given to the type of instruction students have received in the jurisdiction and the adequacy of curricular implementation.

There will be other factors that are of importance in particular jurisdictions. School boards wishing to examine further the results in light of local factors are encouraged to establish their own local interpretation panels.

Absentee Rates

If more than 10% of the eligible students in a jurisdiction did not write the test, the reported averages for that jurisdiction may not accurately represent the true averages. Teacher-assigned marks for students who did not write could be compared with teacher-assigned marks for students who did write. If the averages are the same for the two groups, the reported achievement averages are probably representative. If the averages are different, some estimates can be made of what the achievement averages might have been if all students had written the test. Jurisdictions with high absentee rates may wish to contact the Student Evaluation Branch for assistance in estimating their averages.

Concluding Observations

The provincial average for the total test was 66.3%.

Students had difficulty with questions that required them to:

- perform arithmetical operations involving negative rationals,
- solve problems involving the Theorem of Pythagoras,
- recognize the number of sides in a pentagon, a hexagon, and an octagon,
- solve problems involving surface areas of three-dimensional objects,
- predict the effect of altering specific elements of formulas,
- recognize algebraic terminology,
- factor and find the product of polynomials, and
- solve problems by writing equations in one variable and solving the equations.